Exercise 7.2. Calculate the matrix elements of $\sigma_z \otimes \tau_x$ by forming inner products as we did in Eq. 7.2.

Similar to equation 7.2 we have for $\sigma_z \otimes \tau_x$

$$\sigma_z \otimes \tau_x = \begin{pmatrix} \langle uu|\sigma_z\tau_x|uu\rangle & \langle uu|\sigma_z\tau_x|ud\rangle & \langle uu|\sigma_z\tau_x|du\rangle & \langle uu|\sigma_z\tau_x|dd\rangle \\ \langle ud|\sigma_z\tau_x|uu\rangle & \langle ud|\sigma_z\tau_x|ud\rangle & \langle ud|\sigma_z\tau_x|du\rangle & \langle ud|\sigma_z\tau_x|dd\rangle \\ \langle du|\sigma_z\tau_x|uu\rangle & \langle du|\sigma_z\tau_x|ud\rangle & \langle du|\sigma_z\tau_x|du\rangle & \langle du|\sigma_z\tau_x|dd\rangle \\ \langle dd|\sigma_z\tau_x|uu\rangle & \langle dd|\sigma_z\tau_x|ud\rangle & \langle dd|\sigma_z\tau_x|du\rangle & \langle dd|\sigma_z\tau_x|dd\rangle \end{pmatrix}$$

Let σ_z operate on the left as in equation 7.3. The result is

$$\sigma_z \otimes \tau_x = \begin{pmatrix} \langle uu|\tau_x|uu\rangle & \langle uu|\tau_x|ud\rangle & \langle uu|\tau_x|du\rangle & \langle uu|\tau_x|dd\rangle \\ \langle ud|\tau_x|uu\rangle & \langle ud|\tau_x|ud\rangle & \langle ud|\tau_x|du\rangle & \langle ud|\tau_x|dd\rangle \\ -\langle du|\tau_x|uu\rangle & -\langle du|\tau_x|ud\rangle & -\langle du|\tau_x|du\rangle & -\langle du|\tau_x|dd\rangle \\ -\langle dd|\tau_x|uu\rangle & -\langle dd|\tau_x|ud\rangle & -\langle dd|\tau_x|du\rangle & -\langle dd|\tau_x|dd\rangle \end{pmatrix}$$

Let τ_x operate on the right. Recalling from exercise 7.1 that τ_x flips the direction Bob's spin we have

$$\sigma_z \otimes \tau_x = \begin{pmatrix} \langle uu|ud\rangle & \langle uu|uu\rangle & \langle uu|dd\rangle & \langle uu|du\rangle \\ \langle ud|ud\rangle & \langle ud|uu\rangle & \langle ud|dd\rangle & \langle ud|du\rangle \\ -\langle du|ud\rangle & -\langle du|uu\rangle & -\langle du|dd\rangle & -\langle du|du\rangle \\ -\langle dd|ud\rangle & -\langle dd|uu\rangle & -\langle dd|dd\rangle & -\langle dd|du\rangle \end{pmatrix}$$

Hence

$$\sigma_z \otimes \tau_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$