Exercise 6.4. Use the matrix forms of σ_z, σ_x, and σ_y and the column vectors for $|u\}$ and $|d\}$ to verify Eqs. 6.6. Then, use Eqs. 6.6 and 6.7 to write the equations that were left out of Eqs. 6.8. Use the appendix to check your answers.

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{3.20}$$

$$|u\} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |d\} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Hence

$$\sigma_z|u\} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |u\}$$

$$\sigma_z|d\} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|d\}$$

$$\sigma_x|u\} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |d\}$$

$$\sigma_x|d\} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |u\}$$

$$\sigma_y|u\} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i|d\}$$

$$\sigma_y|d\} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i|u\}$$

For σ_z of product states we have

$$\sigma_z|uu\rangle = \sigma_z|u\rangle \otimes |u\rangle = |u\rangle \otimes |u\rangle = |uu\rangle$$

$$\sigma_z|ud\rangle = \sigma_z|u\rangle \otimes |d\rangle = |u\rangle \otimes |d\rangle = |ud\rangle$$

$$\sigma_z|du\rangle = \sigma_z|d\rangle \otimes |u\rangle = -|d\rangle \otimes |u\rangle = -|du\rangle$$

$$\sigma_z|dd\rangle = \sigma_z|d\rangle \otimes |d\rangle = -|d\rangle \otimes |d\rangle = -|dd\rangle$$

For σ_x of product states we have

$$\sigma_x|uu\rangle = \sigma_x|u\rangle \otimes |u\rangle = |d\rangle \otimes |u\rangle = |du\rangle$$

$$\sigma_x|ud\rangle = \sigma_x|u\rangle \otimes |d\rangle = |d\rangle \otimes |d\rangle = |dd\rangle$$

$$\sigma_x|du\rangle = \sigma_x|d\rangle \otimes |u\rangle = |u\rangle \otimes |u\rangle = |uu\rangle$$

$$\sigma_x|dd\rangle = \sigma_x|d\rangle \otimes |d\rangle = |u\rangle \otimes |d\rangle = |ud\rangle$$

For σ_y of product states we have

$$\sigma_y|uu\rangle = \sigma_y|u\rangle \otimes |u\rangle = i|d\rangle \otimes |u\rangle = i|du\rangle$$

$$\sigma_y|ud\rangle = \sigma_y|u\rangle \otimes |d\rangle = i|d\rangle \otimes |d\rangle = i|dd\rangle$$

$$\sigma_y|du\rangle = \sigma_y|d\rangle \otimes |u\rangle = -i|u\rangle \otimes |u\rangle = -i|uu\rangle$$

$$\sigma_y|dd\rangle = \sigma_y|d\rangle \otimes |d\rangle = -i|u\rangle \otimes |d\rangle = -i|ud\rangle$$

For τ_z of product states we have

$$\tau_z|uu\rangle = |u\rangle \otimes \tau_z|u\rangle = |u\rangle \otimes |u\rangle = |uu\rangle$$

$$\tau_z|ud\rangle = |u\rangle \otimes \tau_z|d\rangle = |u\rangle \otimes -|d\rangle = -|ud\rangle$$

$$\tau_z|du\rangle = |d\rangle \otimes \tau_z|u\rangle = |d\rangle \otimes |u\rangle = |du\rangle$$

$$\tau_z|dd\rangle = |d\rangle \otimes \tau_z|d\rangle = |d\rangle \otimes -|d\rangle = -|dd\rangle$$

For τ_x of product states we have

$$\tau_x|uu\rangle = |u\rangle \otimes \tau_x|u\rangle = |u\rangle \otimes |d\rangle = |ud\rangle$$

$$\tau_x|ud\rangle = |u\rangle \otimes \tau_x|d\rangle = |u\rangle \otimes |u\rangle = |uu\rangle$$

$$\tau_x|du\rangle = |d\rangle \otimes \tau_x|u\rangle = |d\rangle \otimes |d\rangle = |dd\rangle$$

$$\tau_x|dd\rangle = |d\rangle \otimes \tau_x|d\rangle = |d\rangle \otimes |u\rangle = |du\rangle$$

For τ_y of product states we have

$$\tau_y|uu\rangle = |u\rangle \otimes \tau_y|u\rangle = |u\rangle \otimes i|d\rangle = i|ud\rangle$$

$$\tau_y|ud\rangle = |u\rangle \otimes \tau_y|d\rangle = |u\rangle \otimes -i|u\rangle = -i|uu\rangle$$

$$\tau_y|du\rangle = |d\rangle \otimes \tau_y|u\rangle = |d\rangle \otimes i|d\rangle = i|dd\rangle$$

$$\tau_y|dd\rangle = |d\rangle \otimes \tau_y|d\rangle = |d\rangle \otimes -i|u\rangle = -i|du\rangle$$