Exercise 4.2. Prove that if $\mathbf{M}$ and $\mathbf{L}$ are both Hermitian, $i[\mathbf{M}, \mathbf{L}]$ is also Hermitian. Note that the i is important. The commutator is, by itself, not Hermitian.

We have

$$(i[\mathbf{M}, \mathbf{L}])^\dagger = (i\mathbf{ML} - i\mathbf{LM})^\dagger = -i(\mathbf{ML})^\dagger + i(\mathbf{LM})^\dagger$$

Noting that $(\mathbf{ML})^\dagger = \mathbf{L}^\dagger \mathbf{M}^\dagger$ we have

$$(i[\mathbf{M}, \mathbf{L}])^\dagger = -i\mathbf{L}^\dagger \mathbf{M}^\dagger + i\mathbf{M}^\dagger \mathbf{L}^\dagger$$

By hypothesis $\mathbf{M} = \mathbf{M}^\dagger$ and $\mathbf{L} = \mathbf{L}^\dagger$ hence

$$(i[\mathbf{M}, \mathbf{L}])^\dagger = -i\mathbf{LM} + i\mathbf{ML} = i[\mathbf{M}, \mathbf{L}]$$