

Consider a system with the following eigenstates.

$$\begin{aligned}
 |0\rangle &= (1\ 0\ 0\ 0)^\dagger && \text{no electrons} \\
 |1\rangle &= (0\ 1\ 0\ 0)^\dagger && \text{one electron in state } \phi_1 \\
 |2\rangle &= (0\ 0\ 1\ 0)^\dagger && \text{one electron in state } \phi_2 \\
 |3\rangle &= (0\ 0\ 0\ 1)^\dagger && \text{two electrons, one in state } \phi_1, \text{ one in state } \phi_2
 \end{aligned}$$

Then for the wavefunction basis

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

and for  $L = 10^{-9}$  meters we have

$$\hat{E} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.38\text{ eV} & 0 & 0 \\ 0 & 0 & 1.50\text{ eV} & 0 \\ 0 & 0 & 0 & 6.55\text{ eV} \end{pmatrix}$$

Let  $|\xi\rangle$  be the state vector

$$|\xi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

The expected energy is

$$\langle\xi|\hat{E}|\xi\rangle = \frac{0\text{ eV}}{4} + \frac{0.38\text{ eV}}{4} + \frac{1.50\text{ eV}}{4} + \frac{6.55\text{ eV}}{4} = 2.11\text{ eV}$$

For the system we are considering, the result of a single measurement is either 0 eV, 0.38 eV, 1.50 eV, or 6.55 eV. The value 2.11 eV is the expected average across multiple measurements. Recall that a measurement causes the system to exit state  $|\xi\rangle$  and enter an eigenstate  $|0\rangle$ ,  $|1\rangle$ ,  $|2\rangle$ , or  $|3\rangle$  corresponding to the measured eigenvalue. The system must be put back in state  $|\xi\rangle$  before the next measurement.

To use a slot machine analogy, state  $|\xi\rangle$  is like the wheels spinning. Observing the system makes the wheels stop. The stopped wheels are in an eigenstate  $|0\rangle$ ,  $|1\rangle$ ,  $|2\rangle$ , or  $|3\rangle$ . Once they are stopped the wheels don't change, they remain in the same eigenstate. You have to pull the lever to get the wheels spinning again.