15.5.1. Find the commutator $[\hat{\xi}_\lambda, \hat{\pi}_\lambda]$ starting from the operator definitions

$$\hat{\xi}_\lambda \equiv \frac{1}{\sqrt{2}}\left(\hat{a}_\lambda + \hat{a}_\lambda^\dagger\right) \quad \text{and} \quad \hat{\pi}_\lambda \equiv \frac{i}{\sqrt{2}}\left(\hat{a}_\lambda^\dagger - \hat{a}_\lambda\right)$$

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We have

$$\hat{\xi}_\lambda \hat{\pi}_\lambda = \frac{i}{2}\left(\hat{a}_\lambda \hat{a}_\lambda^\dagger - \hat{a}_\lambda \hat{a}_\lambda + \hat{a}_\lambda^\dagger \hat{a}_\lambda^\dagger - \hat{a}_\lambda^\dagger \hat{a}_\lambda\right) \tag{1}$$

and

$$\hat{\pi}_\lambda \hat{\xi}_\lambda = \frac{i}{2}\left(\hat{a}_\lambda^\dagger \hat{a}_\lambda + \hat{a}_\lambda^\dagger \hat{a}_\lambda^\dagger - \hat{a}_\lambda \hat{a}_\lambda - \hat{a}_\lambda \hat{a}_\lambda^\dagger\right) \tag{2}$$

Subtract (2) from (1) to obtain the commutator.

$$[\hat{\xi}_\lambda, \hat{\pi}_\lambda] = i\left(\hat{a}_\lambda \hat{a}_\lambda^\dagger - \hat{a}_\lambda^\dagger \hat{a}_\lambda\right)$$

From

$$[\hat{a}_\lambda, \hat{a}_\lambda^\dagger] = \hat{a}_\lambda \hat{a}_\lambda^\dagger - \hat{a}_\lambda^\dagger \hat{a}_\lambda = 1$$

we have

$$[\hat{\xi}_\lambda, \hat{\pi}_\lambda] = i$$