Arc length

Let $g(t)$ be a parametric function that draws a curve in $\mathbb{R}^n$. The arc length from $g(a)$ to $g(b)$ is given by

$$\int_a^b |g'(t)|\,dt$$

where $|g'(t)|$ is the length of the tangent vector at $g(t)$.

Example 1. Find the length of the curve $y = x^2$ from $x = 0$ to $x = 1$.

```
g = (t,t^2)
defint(abs(d(g,t)),t,0,1)
```

$\frac{1}{2}\,5^{1/2} - \frac{1}{4}\log(2) + \frac{1}{4}\log(2\,5^{1/2} + 4)$

```
float
```

1.47894

As expected, the result is greater than $\sqrt{2} \approx 1.414$, the length of a straight line from $(0, 0)$ to $(1, 1)$.

The following script does a discrete computation of the arc length by dividing the curve into 100 pieces.

```
g(t) = (t,t^2)
h(k) = abs(g(k/100.0) - g((k-1)/100.0))
sum(k,1,100,h(k))
```

1.47894

As expected, the discrete result matches the analytic result.

Example 2. Find the length of the curve $y = x^{3/2}$ from the origin to $x = \frac{4}{3}$.

```
g = (t,t^(3/2))
defint(abs(d(g,t)),t,0,4/3)
```

$\frac{56}{27}$