

(9.1) Deduce that the generators of the translation operator are given by

$$\hat{\mathbf{p}} = -\frac{1}{i} \left. \frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} \right|_{\mathbf{a}=\mathbf{0}} \quad (9.51)$$

From page 81 we have

$$\hat{U}(\mathbf{a}) = \exp(-i\hat{\mathbf{p}} \cdot \mathbf{a})$$

Hence

$$\frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} = -i\hat{\mathbf{p}} \exp(-i\hat{\mathbf{p}} \cdot \mathbf{a})$$

Then for $\mathbf{a} = \mathbf{0}$ we have

$$\left. \frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} \right|_{\mathbf{a}=\mathbf{0}} = -i\hat{\mathbf{p}}$$