

(7.2) Consider a massive scalar field  $\phi(x)$  coupled to a source  $J(x)$ , described by the Lagrangian of eqn 7.10. Show that the equations of motion are those of eqn 7.11.

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$$\mathcal{L} = \frac{1}{2}[\partial_\mu\phi(x)]^2 - \frac{1}{2}m^2[\phi(x)]^2 + J(x)\phi(x) \quad (7.10)$$

$$(\partial_\mu\partial^\mu + m^2)\phi(x) = J(x) \quad (7.11)$$

For the Lagrangian  $\mathcal{L}$  given in (7.10) we have

$$\frac{\partial\mathcal{L}}{\partial\phi} = -m^2\phi(x) + J(x)$$

and

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \partial^\mu\phi(x)$$

Then by the Euler-Lagrange equation we have

$$\partial_\mu\partial^\mu\phi(x) + m^2\phi(x) - J(x) = 0$$

which is equivalent to (7.11).