(5.1) If the Lagrangian does depend explicitly on time, then

$$\frac{dL}{dt} = \frac{\partial L}{\partial t} + \frac{\partial L}{\partial x_i}\dot{x}_i + \frac{\partial L}{\partial \dot{x}_i}\ddot{x}_i \tag{5.57}$$

In this case show that

$$\frac{\partial L}{\partial t} = -\frac{dH}{dt} \tag{5.58}$$

Recall that d/dt is the total time derivative.

Consider equation (5.6).

$$H = p_i\dot{q}_i - L \tag{5.6}$$

Applying the total time derivative to H we have

$$\frac{dH}{dt} = \frac{\partial(p_i\dot{q}_i)}{\partial p_i}\frac{dp_i}{dt} + \frac{\partial(p_i\dot{q}_i)}{\partial \dot{q}_i}\frac{d\dot{q}_i}{dt} - \frac{dL}{dt}$$

Hence

$$\frac{dH}{dt} = \dot{p}_i\dot{q}_i + p_i\ddot{q}_i - \frac{dL}{dt}$$

Substitute

$$p_i = \frac{\partial L}{\partial \dot{x}_i} \qquad q_i = x_i$$

to obtain

$$\frac{dH}{dt} = \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}\right)\dot{x}_i + \frac{\partial L}{\partial \dot{x}_i}\ddot{x}_i - \frac{dL}{dt} \tag{1}$$

Consider the following Euler-Lagrange equation.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = \frac{\partial L}{\partial x_i} \tag{2}$$

Substitute (2) into (1) to obtain

$$\frac{dH}{dt} = \frac{\partial L}{\partial x_i}\dot{x}_i + \frac{\partial L}{\partial \dot{x}_i}\ddot{x}_i - \frac{dL}{dt} = -\frac{\partial L}{\partial t}$$