

(37.2) Show that both choices of sign $\mathbf{K} = \pm \frac{i\boldsymbol{\sigma}}{2}$ obey the necessary commutation relations.

These are the commutation relations from page 336.

$$\begin{aligned} [J^i, K^j] &= i\varepsilon^{ijk} K^k \\ [K^i, K^j] &= -i\varepsilon^{ijk} J^k \end{aligned}$$

On page 337 we are given

$$\mathbf{J} = \frac{1}{2}\boldsymbol{\sigma}$$

and

$$[\sigma^i, \sigma^j] = 2i\varepsilon^{ijk} \sigma^k \tag{1}$$

For the first commutation relation we have

$$[J^i, \pm K^j] = \left[\frac{1}{2}\sigma^i, \pm \frac{i}{2}\sigma^j \right] = \pm \frac{i}{4}[\sigma^i, \sigma^j]$$

By equation (1)

$$\pm \frac{i}{4}[\sigma^i, \sigma^j] = \pm \frac{i}{4}(2i\varepsilon^{ijk} \sigma^k) = \mp \frac{1}{2}\varepsilon^{ijk} \sigma^k = \pm i\varepsilon^{ijk} K^k$$

For the second commutation relation we have

$$[K^i, K^j] = \left[\frac{i}{2}\sigma^i, \frac{i}{2}\sigma^j \right] = \left[-\frac{i}{2}\sigma^i, -\frac{i}{2}\sigma^j \right] = -\frac{1}{4}[\sigma^i, \sigma^j]$$

By equation (1)

$$-\frac{1}{4}[\sigma^i, \sigma^j] = -\frac{1}{4}(2i\varepsilon^{ijk} \sigma^k) = -i\varepsilon^{ijk} J^k$$