(1.5) For a three-dimensional elastic medium, the potential energy is

$$V = \frac{\mathcal{T}}{2} \int d^3x \, (\nabla\psi)^2 \tag{1.46}$$

and the kinetic energy is

$$T = \frac{\rho}{2} \int d^3x \left( \frac{\partial\psi}{\partial t} \right)^2 \tag{1.47}$$

Use these results, and the functional derivative approach, to show that $\psi$ obeys the wave equation

$$\nabla^2\psi = \frac{1}{v^2} \frac{\partial^2\psi}{\partial t^2}$$

where $v$ is the velocity of the wave.

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By equation (1.19)

$$\frac{\delta V}{\delta\psi} = -\mathcal{T}\nabla^2\psi \qquad \frac{\delta T}{\delta\psi} = -\rho\frac{\partial^2\psi}{\partial t^2}$$

By equation (1.22)

$$\mathcal{T}\nabla^2\psi = \rho\frac{\partial^2\psi}{\partial t^2}$$

Then for $v^2 = \mathcal{T}/\rho$ we have

$$v^2\nabla^2\psi = \frac{\partial^2\psi}{\partial t^2}$$