

(1.4) Show that

$$\frac{\delta\phi(x)}{\delta\phi(y)} = \delta(x - y) \quad (1.44)$$

and

$$\frac{\delta\dot{\phi}(t)}{\delta\phi(t_0)} = \frac{d}{dt}\delta(t - t_0) \quad (1.45)$$

Let F be the trivial functional $F[\phi] = \phi(x)$.

$$\frac{\delta F}{\delta\phi(y)} = \lim_{\epsilon \rightarrow 0} \frac{\phi(x) + \epsilon\delta(x - y) - \phi(x)}{\epsilon} = \delta(x - y)$$

Show (1.45).

$$\frac{\delta\dot{\phi}(t)}{\delta\phi(t_0)} = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \left(\frac{d(\phi(t) - \epsilon\delta(t - t_0))}{dt} - \frac{d\phi(t)}{dt} \right) = \frac{d}{dt}\delta(t - t_0)$$