9-6. Show, using sine and cosine modes and real variables, that this expression using complex variables is indeed correct (cf. problem 8-4).

$$\Phi_0 = \exp\left(-\sum_{\mathbf{k}} \frac{kc}{2\hbar}\left(\bar{a}^*_{1,\mathbf{k}}\bar{a}_{1,\mathbf{k}} + \bar{a}^*_{2,\mathbf{k}}\bar{a}_{2,\mathbf{k}}\right)\right) \tag{9.43}$$

Let

$$\begin{aligned}
\bar{a}^c_{1,\mathbf{k}} &= \frac{1}{\sqrt{2}}(\bar{a}_{1,\mathbf{k}} + \bar{a}^*_{1,\mathbf{k}}) \\
\bar{a}^s_{1,\mathbf{k}} &= \frac{i}{\sqrt{2}}(\bar{a}_{1,\mathbf{k}} - \bar{a}^*_{1,\mathbf{k}})
\end{aligned}$$

$$\begin{aligned}
\bar{a}^c_{2,\mathbf{k}} &= \frac{1}{\sqrt{2}}(\bar{a}_{2,\mathbf{k}} + \bar{a}^*_{2,\mathbf{k}}) \\
\bar{a}^s_{2,\mathbf{k}} &= \frac{i}{\sqrt{2}}(\bar{a}_{2,\mathbf{k}} - \bar{a}^*_{2,\mathbf{k}})
\end{aligned}$$

Then

$$\begin{aligned}
\bar{a}_{1,\mathbf{k}} &= \frac{1}{\sqrt{2}}(\bar{a}^c_{1,\mathbf{k}} - i\bar{a}^s_{1,\mathbf{k}}) \\
\bar{a}_{2,\mathbf{k}} &= \frac{1}{\sqrt{2}}(\bar{a}^c_{2,\mathbf{k}} - i\bar{a}^s_{2,\mathbf{k}})
\end{aligned}$$

It follows that

$$\begin{aligned}
\bar{a}^*_{1,\mathbf{k}}\bar{a}_{1,\mathbf{k}} &= \frac{1}{2}(\bar{a}^c_{1,\mathbf{k}})^2 + \frac{1}{2}(\bar{a}^s_{1,\mathbf{k}})^2 \\
\bar{a}^*_{2,\mathbf{k}}\bar{a}_{2,\mathbf{k}} &= \frac{1}{2}(\bar{a}^c_{2,\mathbf{k}})^2 + \frac{1}{2}(\bar{a}^s_{2,\mathbf{k}})^2
\end{aligned} \tag{1}$$

Substitute (1) into (9.43).

$$\Phi_0 = \exp\left(-\sum_{\mathbf{k}} \frac{kc}{4\hbar}\left((\bar{a}^c_{1,\mathbf{k}})^2 + (\bar{a}^s_{1,\mathbf{k}})^2 + (\bar{a}^c_{2,\mathbf{k}})^2 + (\bar{a}^s_{2,\mathbf{k}})^2\right)\right)$$