7-7. Show that for any quadratic action

$$\langle x(t)\rangle = \bar{x}(t)\langle 1\rangle \tag{7.57}$$

From p. 59, $\bar{x}(t)$ is the classical path.

The following solution is similar to problem 2-1.

The general quadratic Lagrangian is given by equation (3.44).

$$L = a(t)\dot{x}^2 + b(t)\dot{x}x + c(t)x^2 + d(t)\dot{x} + e(t)x + f(t) \tag{3.44}$$

It follows that

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 2\frac{d}{dt}a(t)\dot{x} + 2a(t)\ddot{x} + \frac{d}{dt}b(t)x + b(t)\dot{x} + \frac{d}{dt}d(t) \tag{1}$$

and

$$\frac{\partial L}{\partial x} = b(t)\dot{x} + 2c(t)x + e(t) \tag{2}$$

Consider equation (2.7) which determines the classical path $\bar{x}(t)$.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \tag{2.7}$$

By equation (2.7) and (1) and (2) we have

$$\ddot{x} = 0$$

Hence velocity $\dot{x}$ is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a}$$

It follows that

$$\bar{x}(t) = x_a + \frac{t}{t_b - t_a}(x_b - x_a) \tag{3}$$

Consider equation (7.56).

$$\langle x(t)\rangle = \left(x_a + \frac{t}{T}(x_b - x_a)\right)\langle 1\rangle \tag{7.56}$$

Substitute (3) into (7.56) to obtain (7.57).

$$\langle x(t)\rangle = \bar{x}(t)\langle 1\rangle \tag{7.57}$$