5-8. Note that equation (5.44) implies $G_A^*(x,x') = G_A(x',x)$. With this in mind show that for any two wave functions $g(x)$ and $f(x)$, both of which approach 0 as x goes to $\pm\infty$,

$$\int_{-\infty}^{\infty} g^*(x)\mathcal{A}f(x)\,dx = \int_{-\infty}^{\infty} \big(\mathcal{A}g(x)\big)^* f(x)\,dx \tag{5.47}$$

Any operator, such as $\mathcal{A}$, for which equation (5.47) holds is called *hermitian* (see equation 4.30).

From equations (5.45) and (5.43)

$$\mathcal{A}f(x) = \int_{-\infty}^{\infty} G_A(x,x')f(x')\,dx' \tag{1}$$

Hence

$$\int_{-\infty}^{\infty} g^*(x)\mathcal{A}f(x)\,dx = \int_{-\infty}^{\infty} g^*(x)\left(\int_{-\infty}^{\infty} G_A(x,x')f(x')\,dx'\right)dx$$

Combine factors.

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_A(x,x')g^*(x)f(x')\,dx'\,dx$$

By equation (5.44) substitute $G_A^*(x',x)$ for $G_A(x,x')$.

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_A^*(x',x)g^*(x)f(x')\,dx'\,dx$$

By complex conjugation of a product

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \big(G_A(x',x)g(x)\big)^* f(x')\,dx'\,dx$$

Factor out $f(x')\,dx'$.

$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} \big(G_A(x',x)g(x)\big)^*\,dx\right) f(x')\,dx'$$

Then by equation (1)

$$= \int_{-\infty}^{\infty} \big(\mathcal{A}g(x')\big)^* f(x')\,dx'$$