5-5. Assume that the function $f(x, y, z, \dots)$ can be represented by

$$f(x, y, z, \dots) = \sum_a \sum_b \sum_c \cdots F'_{a,b,c,\dots} \chi_{a,b,c,\dots}(x, y, z, \dots) \tag{5.38}$$

By substituting this relation into equation (5.36), and using the orthogonal properties of χ as defined by equation (5.35), show that $F'_{a,b,c,\dots} = F_{a,b,c,\dots}$.

Take (5.38) and prime the indices.

$$f(x, y, z, \dots) = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots) \tag{1}$$

This is equation (5.36).

$$F_{a,b,c,\dots} = \int_{-\infty}^{\infty} \chi^*_{a,b,c,\dots}(x) f(x)\, dx \tag{5.36}$$

Substitute the right-hand side of (1) for $f(x)$ in (5.36).

$$\begin{aligned} F_{a,b,c,\dots} = \int_{\mathbb{R}^n} & \chi^*_{a,b,c,\dots}(x, y, z, \dots) \\ & \times \left(\sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots) \right) dx\, dy\, dz\, \cdots \end{aligned}$$

Combine factors.

$$\begin{aligned} F_{a,b,c,\dots} = \int_{\mathbb{R}^n} & \sum_{a'} \sum_{b'} \sum_{c'} \cdots \\ & \chi^*_{a,b,c,\dots}(x, y, z, \dots) F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots)\, dx\, dy\, dz\, \cdots \end{aligned}$$

Interchange the order of integration and summation.

$$\begin{aligned} F_{a,b,c,\dots} = \sum_{a'} \sum_{b'} \sum_{c'} & \cdots \int_{\mathbb{R}^n} \\ & \chi^*_{a,b,c,\dots}(x, y, z, \dots) F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots)\, dx\, dy\, dz\, \cdots \end{aligned}$$

Factor out F'.

$$F_{a,b,c,\dots} = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots}$$
$$\times \left(\int_{\mathbb{R}^n} \chi^*_{a,b,c,\dots}(x,y,z,\dots) \chi_{a',b',c',\dots}(x,y,z,\dots)\, dx\, dy\, dz\, \cdots \right)$$

By equation (5.35) the integral becomes a product of delta functions.

$$F_{a,b,c,\dots} = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \delta(a-a')\delta(b-b')\delta(c-c') \cdots$$

Hence for $F_{a,b,c,\dots} \neq 0$ we must have $a = a'$, $b = b'$, etc. Therefore

$$F_{a,b,c,\dots} = F'_{a',b',c',\dots} = F'_{a,b,c,\dots}$$