5-4. Suppose the wave function for a system is $\psi(x)$ at time t_a. Suppose further that the behavior of the system described by the kernel $K(x_b, t_b, x_a, t_a)$ for motions in the interval $t_b \geq t \geq t_a$. Show that the probability that the system is found to be in state $\chi(x)$ at time t_b is given by the square of the integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, t_b, x_a, t_a) \psi(x_a)\, dx_a\, dx_b \tag{1}$$

We call this integral the *transition amplitude* to go from state $\psi(x)$ to state $\chi(x)$.

From equation (3.42)

$$\psi(x_b, t_b) = \int_{-\infty}^{\infty} K(x_b, t_b, x_a, t_a) \psi(x_a, t_a)\, dx_a$$

Hence the integral over x_a in (1) is equivalent to $\psi(x_b)$.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, t_b, x_a, t_a) \psi(x_a)\, dx_a\, dx_b = \int_{-\infty}^{\infty} \chi^*(x_b) \psi(x_b)\, dx_b$$

Then by equation (5.32)

$$P(\mathrm{X}) = \left| \int_{-\infty}^{\infty} \chi^*(x_b) \psi(x_b)\, dx_b \right|^2$$

where X is upper-case χ.