

4-8. Show from the fact that H is hermitian that E is real. Hint: Choose $f = g = \phi$ in equation (4.30).

The Hamiltonian H is an eigenfunction with corresponding eigenvalue E .

$$H\phi(x) = E\phi(x) \tag{4.42}$$

Since H is hermitian we have from equation (4.30)

$$\int_{-\infty}^{\infty} (Hg)^* f dx = \int_{-\infty}^{\infty} g^* (Hf) dx \tag{4.30}$$

Substitute ϕ into f and g .

$$\int_{-\infty}^{\infty} (H\phi)^* \phi dx = \int_{-\infty}^{\infty} \phi^* (H\phi) dx$$

Replace H with eigenvalue E .

$$\int_{-\infty}^{\infty} (E\phi)^* \phi dx = \int_{-\infty}^{\infty} \phi^* E\phi dx$$

Since E is a constant it can be factored out of the integrands.

$$E^* \int_{-\infty}^{\infty} \phi^* \phi dx = E \int_{-\infty}^{\infty} \phi^* \phi dx$$

The integrals are identical hence

$$E^* = E$$