

3-7. Using equations (3.52) and (2.31), express  $F(t+s)$  in terms of  $F(t)$  and  $F(s)$ , where  $t = t_b - t_c$  and  $s = t_c - t_a$ .

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From equation (3.52) and  $t + s = t_b - t_a$

$$K(b, a) = F(t+s) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t+s)}\right) \quad (1)$$

From equation (2.31)

$$K(b, a) = \int_{-\infty}^{\infty} K(b, c)K(c, a) dx_c$$

Hence

$$K(b, a) = F(t)F(s) \int_{-\infty}^{\infty} \exp\left(\frac{im(x_b - x_c)^2}{2\hbar t}\right) \exp\left(\frac{im(x_c - x_a)^2}{2\hbar s}\right) dx_c$$

Solve the integral.

$$K(b, a) = F(t)F(s) \left(\frac{2\pi i\hbar ts}{m(t+s)}\right)^{1/2} \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t+s)}\right) \quad (2)$$

Equating (1) with (2) causes the exponentials to cancel leaving

$$F(t+s) = F(t)F(s) \left(\frac{2\pi i\hbar ts}{m(t+s)}\right)^{1/2} \quad (3)$$

Show that if

$$F(t) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} f(t)$$

then the new function  $f(t)$  must satisfy

$$f(t+s) = f(t)f(s)$$

By substitution

$$F(t+s) = \left(\frac{m}{2\pi i\hbar(t+s)}\right)^{1/2} f(t+s)$$

and

$$F(t)F(s) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \left(\frac{m}{2\pi i\hbar s}\right)^{1/2} f(t)f(s)$$

Then by (3) we have

$$\begin{aligned} \left(\frac{m}{2\pi i\hbar(t+s)}\right)^{1/2} f(t+s) = \\ \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \left(\frac{m}{2\pi i\hbar s}\right)^{1/2} f(t)f(s) \left(\frac{2\pi i\hbar ts}{m(t+s)}\right)^{1/2} \end{aligned}$$

The coefficients cancel leaving

$$f(t+s) = f(t)f(s) \tag{4}$$