

3-6. Since the free-particle lagrangian is quadratic, show that

$$K(b, a) = F(t_b, t_a) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right)$$

and give an argument to show that  $F$  can depend only on the difference  $F(t_b - t_a)$ .

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From equation (3.51)

$$K(b, a) = \exp\left(\frac{i}{\hbar}S_{cl}(b, a)\right) F(t_b, t_a)$$

From problem (2.1)

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a}$$

Hence by substitution of  $S_{cl}$

$$K(b, a) = F(t_b, t_a) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right)$$

The argument for why  $F$  can only depend on  $t_b - t_a$  is invariance under time translation. For any constant  $s$  we should have

$$F(t_b, t_a) = F(t_b + s, t_a + s)$$

For example, we should have

$$F(1, 0) = F(3, 2)$$

Hence  $F$  can only depend on the difference  $t_b - t_a$  and not the specific values of  $t_a$  and  $t_b$ .