3-1. The probability that a particle arrives at the point b is by definition proportional to the absolute square of the kernel $K(b, a)$. For the free-particle kernel of equation (3.3) this is

$$P(b)\,dx = \frac{m}{2\pi\hbar(t_b - t_a)}\,dx \tag{3.6}$$

Clearly this is a relative probability, since the integral over the complete range of x diverges. What does the particular normalization mean? Show that this corresponds to a classical picture in which a particle starts from the point a with all momenta equally likely. Show that the corresponding relative probability that the momentum of the particle lies in the range dp is $dp/2\pi\hbar$.

Compute the normalization constant C.

$$\begin{aligned} C &= \int_{x_a}^{x_b} P(b)\,dx \\ &= \frac{m}{2\pi\hbar(t_b - t_a)}\,x\Big|_{x_a}^{x_b} \\ &= \frac{m}{2\pi\hbar}\left(\frac{x_b - x_a}{t_b - t_a}\right) \end{aligned}$$

From $v = (x_b - x_a)/(t_b - t_a)$ and $p = mv$ we have

$$C = \frac{p}{2\pi\hbar}$$

Hence diverging normalization C corresponds to unrestricted momentum p.

From $v = p/m$ we have

$$\frac{x_b - x_a}{t_b - t_a} + \frac{dx}{t_b - t_a} = \frac{p}{m} + \frac{dp}{m}$$

It follows that

$$dx = \frac{dp}{m}(t_b - t_a)$$

Multiply both sides by $P(b)$.

$$P(b)\,dx = \frac{dp}{2\pi\hbar}$$