

Translation

Let K_1 , K_2 , and K_3 be translation operators.

$$K_1 = \frac{1}{\hbar}P_1, \quad K_2 = \frac{1}{\hbar}P_2, \quad K_3 = \frac{1}{\hbar}P_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon K_3 - \frac{1}{2}\epsilon^2 K_3^2$$

1. Show that to order ϵ^2

$$\begin{aligned}U^{-1}X_1U &= X_1 \\U^{-1}X_2U &= X_2 \\U^{-1}X_3U &= X_3 + \epsilon\end{aligned}$$

2. Show that to order ϵ^2

$$\begin{aligned}U^{-1}P_1U &= P_1 \\U^{-1}P_2U &= P_2 \\U^{-1}P_3U &= P_3\end{aligned}$$

3. Show that to order ϵ^2

$$\begin{aligned}U^{-1}L_1U &= L_1 - \epsilon P_2 \\U^{-1}L_2U &= L_2 + \epsilon P_1 \\U^{-1}L_3U &= L_3\end{aligned}$$

4. Show that to order ϵ^2

$$U^{-1}HU = H$$

where

$$H = \frac{1}{2m} (P_1^2 + P_2^2 + P_3^2)$$