Time-independent Schrodinger equation

Consider the Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = H\Psi(x,t)$$

Let $\Psi(x,t)$ be the wavefunction

$$\Psi(x,t) = \sum_k \xi_k(t)\psi_k(x)$$

where

$$\xi_k(t) = \exp\left(-\frac{i}{\hbar}E_k t\right)$$

Then for all k we have for the time derivative

$$i\hbar\frac{d}{dt}\xi_k(t)\psi_k(x) = E_k\xi_k(t)\psi_k(x)$$

By the Schrodinger equation the time derivative connects to the Hamiltonian as

$$E_k\xi_k(t)\psi_k(x) = H\left[\xi_k(t)\psi_k(x)\right]$$

Let H be independent of t. Then by linearity of H the Hamiltonian factors as

$$E_k\xi_k(t)\psi_k(x) = \xi_k(t)H\psi_k(x)$$

The $\xi_k(t)$ cancel leaving the time-independent Schrodinger equation

$$E_k\psi_k(x) = H\psi_k(x)$$

By linearity of H we also have

$$\sum_k E_k\psi_k(x) = H\sum_k \psi_k(x)$$

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