Spin part 4

We seek to construct spin state $|s\rangle$ from spin measurements $\langle S_x\rangle$, $\langle S_y\rangle$, and $\langle S_z\rangle$.

Let

$$x = \frac{2}{\hbar}\langle S_x\rangle$$
$$y = \frac{2}{\hbar}\langle S_y\rangle$$
$$z = \frac{2}{\hbar}\langle S_z\rangle$$

By half-angle identities and $z = \cos\theta$ we have

$$\cos(\theta/2) = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+z}{2}}$$
$$\sin(\theta/2) = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-z}{2}}$$

Recalling that

$$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}$$
$$\sin\phi = \frac{y}{\sqrt{x^2+y^2}}$$

we have

$$\exp(i\phi) = \cos\phi + i\sin\phi = \frac{x+iy}{\sqrt{x^2+y^2}}$$

Hence

$$|s\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1+z}{2}} \\ \sqrt{\frac{1-z}{2}}\frac{x+iy}{\sqrt{x^2+y^2}} \end{pmatrix} \tag{1}$$

Eigenmath script