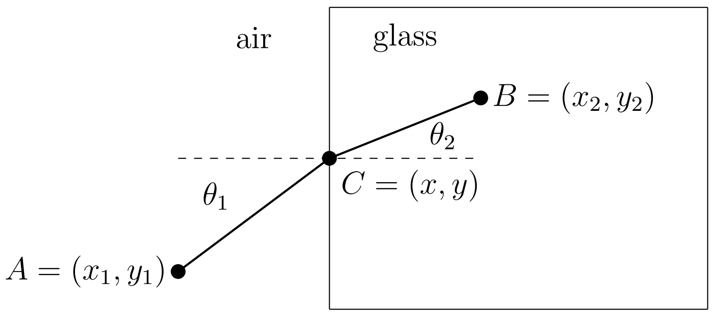
Snell's law

A light ray travels from A in air to B in glass by going through C at the boundary.



Let d_1 be the distance from A to C and let d_2 be the distance from C to B.

$$d_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}, \quad d_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

Let v_1 be the velocity of light through air and let v_2 be the velocity of light through glass. Then the time t to go from A to B is

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2}$$

Differentiate t with respect to y and set the result to zero to find y that minimizes t. (The x coordinate of C is fixed by the boundary between air and glass.)

$$\frac{dt}{dy} = \frac{y - y_1}{v_1 d_1} + \frac{y - y_2}{v_2 d_2} = 0$$

Rewrite as

$$\frac{y - y_1}{v_1 d_1} = \frac{y_2 - y}{v_2 d_2}$$

Hence

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \tag{1}$$

Multiply equation (1) by c to obtain

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where n_1 and n_2 are the refractive indices

$$n_1 = \frac{c}{v_1}, \quad n_2 = \frac{c}{v_2}$$