Scattering part 4

From the previous section we have

$$f(\theta) = -\frac{2m\beta}{p^2 + \epsilon^2\hbar^2}$$

where $V(r)$ is the Yukawa potential

$$V(r) = \frac{\beta}{r}\exp(-\epsilon r)$$

For the Coulomb potential we have

$$V(r) = -\frac{q_1 q_2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Z\alpha\hbar c}{r}$$

Hence for the Coulomb potential substitute $\epsilon = 0$ and $\beta = -Z\alpha\hbar c$ to obtain

$$f(\theta) = \frac{2mZ\alpha\hbar c}{p^2}$$

Substitute

$$p^2 = 4mE(1 - \cos\theta)$$

to obtain

$$f(\theta) = \frac{Z\alpha\hbar c}{2E(1 - \cos\theta)}$$

Hence the Rutherford cross section formula

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \left[\frac{Z\alpha\hbar c}{2E(1 - \cos\theta)}\right]^2$$

Noting that

$$(1 - \cos\theta)^2 = 4\sin^4(\theta/2)$$

we can also write

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z\alpha\hbar c}{2E}\right)^2 \frac{1}{4\sin^4(\theta/2)}$$

Eigenmath script