Scattering part 3

From the previous section we have

$$f(\theta) = -\frac{2m}{\hbar p}\int_0^\infty \sin\left(\frac{pr}{\hbar}\right) V(r)\, r\, dr$$

where p is momentum transfer

$$p = \sqrt{4mE(1-\cos\theta)}$$

Find $f(\theta)$ for the Yukawa potential

$$V(r) = \frac{\beta}{r}\exp(-\epsilon r)$$

By the exponential form of the sine function we have

$$f(\theta) = -\frac{m\beta}{\hbar p}\int_0^\infty \left[i\exp\left(-\frac{ipr}{\hbar}-\epsilon r\right) - i\exp\left(\frac{ipr}{\hbar}-\epsilon r\right)\right] dr$$

Solve the integral.

$$f(\theta) = -\frac{m\beta}{\hbar p}\left[\frac{i}{-ip/\hbar-\epsilon}\exp\left(-\frac{ipr}{\hbar}-\epsilon r\right) - \frac{i}{ip/\hbar-\epsilon}\exp\left(\frac{ipr}{\hbar}-\epsilon r\right)\right]_{r=0}^{r=\infty}$$

Evaluate the limits. The exponentials vanish at the upper limit.

$$f(\theta) = 0 - f(\theta)\bigg|_{r=0} = \frac{m\beta}{\hbar p}\left(\frac{i}{-ip/\hbar-\epsilon} - \frac{i}{ip/\hbar-\epsilon}\right)$$

The fractions rationalize as

$$\frac{i}{-ip/\hbar-\epsilon} - \frac{i}{ip/\hbar-\epsilon} = -\frac{2\hbar p}{p^2+\epsilon^2\hbar^2}$$

Hence for the Yukawa potential

$$f(\theta) = -\frac{2m\beta}{p^2+\epsilon^2\hbar^2}$$

Eigenmath script