Scattering part 1

Let $\psi(r,\theta,\phi)$ be the Born approximation

$$\psi(r,\theta,\phi) = \frac{e^{ikr}}{r} f(\theta,\phi)$$

where

$$k = \frac{\sqrt{2mE}}{\hbar}$$

and $f(\theta,\phi)$ is the scattering amplitude

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2} \int \exp\left(\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right) V(\mathbf{x})\, d\mathbf{x}$$

Born approximation density is

$$|\psi(r,\theta,\phi)|^2 = \frac{|f(\theta,\phi)|^2}{r^2}$$

Cross sections are completely determined by the scattering amplitude.

$$\frac{d\sigma}{d\Omega} = |\psi(r,\theta,\phi)|^2\, r^2 = |f(\theta,\phi)|^2$$

For example, for Coulomb potentials we have

$$f(\theta,\phi) = \frac{Z\alpha\hbar c}{2E(1-\cos\theta)}$$

Hence the Rutherford cross section

$$\frac{d\sigma}{d\Omega} = \left[\frac{Z\alpha\hbar c}{2E(1-\cos\theta)}\right]^2$$

Verify dimensions of exponential.

$$\frac{\sqrt{2mE}}{\hbar}\, r = \frac{\sqrt{[\mathrm{kg}]\,[\mathrm{J}]}}{[\mathrm{J\,s}]}\,[\mathrm{m}] = [1]$$

Verify dimensions of cross section.

$$\left(\frac{Z\alpha\hbar c}{2E}\right)^2 = \left(\frac{[\mathrm{J\,s}]\,[\mathrm{m\,s^{-1}}]}{[\mathrm{J}]}\right)^2 = [\mathrm{m}^2]$$

Eigenmath script