Relativistic energy

Start with relativistic energy E.

$$E = \sqrt{(mc^2)^2 + (pc)^2}$$

Substitute $p = mv$.

$$E = \sqrt{(mc^2)^2 + (mvc)^2}$$

Expand E as a Taylor series in v.

$$E = mc^2 + \frac{1}{2}mv^2 - \frac{mv^4}{8c^2} + \frac{mv^6}{16c^4} - \frac{5mv^8}{128c^6} + \cdots$$

Let v/c vanish for $v \ll c$.

$$E = mc^2 + \frac{1}{2}mv^2$$

Hence the classical kinetic energy

$$E = \frac{1}{2}mv^2$$

Eigenmath script