Orbital angular momentum 1

These are the orbital angular momentum operators in rectangular coordinates.

$$L_1 = X_2 P_3 - X_3 P_2$$
$$L_2 = X_3 P_1 - X_1 P_3$$
$$L_3 = X_1 P_2 - X_2 P_1$$

We also have

$$\mathbf{L} = \mathbf{X} \times \mathbf{P} = \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix}$$
$$L^2 = |\mathbf{L}|^2 = {L_1}^2 + {L_2}^2 + {L_3}^2$$

Let

$$X_j = x_j, \quad P_j = -i\hbar \frac{\partial}{\partial x_j}$$

Verify the following equations where $\psi \equiv \psi(x_1, x_2, x_3, t)$.

$$[\mathbf{X}, L_3]\psi = \begin{pmatrix} -i\hbar X_2 \\ i\hbar X_1 \\ 0 \end{pmatrix} \psi \tag{1}$$

$$[\mathbf{P}, L_3]\psi = \begin{pmatrix} -i\hbar P_2 \\ i\hbar P_1 \\ 0 \end{pmatrix} \psi \tag{2}$$

$$[\mathbf{L}, L^2]\psi = 0 \tag{3}$$

$$\mathbf{L} \times \mathbf{L}\psi = i\hbar \mathbf{L}\psi \tag{4}$$

$$(\mathbf{P} \times \mathbf{L} + \mathbf{L} \times \mathbf{P})\psi = 2i\hbar \mathbf{P}\psi \tag{5}$$

$$\tfrac{1}{2}(\mathbf{P} \times \mathbf{L} - \mathbf{L} \times \mathbf{P})\psi = (\mathbf{P} \times \mathbf{L} - i\hbar \mathbf{P})\psi \tag{6}$$

$$[L_i, P_j]\psi = i\hbar \sum_k \epsilon_{ijk} P_k \psi \tag{7}$$