

Maxwell equations

Let $\mathbf{E}$ be the electric field

$$\mathbf{E} = E_0 \sin(kz - \omega t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and $\mathbf{B}$ the magnetic field

$$\mathbf{B} = \frac{E_0}{c} \sin(kz - \omega t) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

where $k = \omega/c$. Verify $\mathbf{E}$ and $\mathbf{B}$ are solutions to the free-field Maxwell equations

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{E}$$

Electric field $\mathbf{E}$ is measured in Newtons per Coulomb.

$$E_0 = \frac{[\text{N}]}{[\text{C}]}$$

Magnetic field $\mathbf{B}$ is measured in Tesla.

$$\frac{E_0}{c} = \frac{[\text{N}]}{[\text{C}]} \frac{[\text{s}]}{[\text{m}]} = [\text{T}]$$

Let $\mathbf{A}$ be the magnetic vector potential

$$\mathbf{A} = -\frac{E_0}{\omega} \cos(kz - \omega t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Verify

$$\mathbf{E} = -\frac{\partial}{\partial t} \mathbf{A}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Magnetic vector potential $\mathbf{A}$ is measured in Tesla meters.

$$\frac{E_0}{\omega} = \frac{[\text{N}]}{[\text{C}]} [\text{s}] = [\text{T m}]$$

Eigenmath script