l'Hôpital's rule

Find the following limit.

$$y = \lim_{n\to\infty} \left(1 - \frac{\alpha}{n^2}\right)^n$$

Start by rewriting as limit of exponential.

$$y = \lim_{n\to\infty} \exp\left[n \log\left(1 - \frac{\alpha}{n^2}\right)\right]$$

By composition limit law rewrite as exponential of limit.

$$y = \exp\left[\lim_{n\to\infty} n \log\left(1 - \frac{\alpha}{n^2}\right)\right]$$

Make n a denominator.

$$y = \exp\left[\lim_{n\to\infty} \frac{\log\left(1 - \frac{\alpha}{n^2}\right)}{\frac{1}{n}}\right]$$

Both numerator and denominator vanish in the limit hence use l'Hôpital's rule.

$$y = \exp\left[\lim_{n\to\infty} \frac{\frac{d}{dn}\log\left(1 - \frac{\alpha}{n^2}\right)}{\frac{d}{dn}\frac{1}{n}}\right] = \exp\left(\lim_{n\to\infty} \frac{\frac{2\alpha}{n^3-\alpha n}}{-\frac{1}{n^2}}\right)$$

Hence

$$y = \exp\left(\lim_{n\to\infty} \frac{2\alpha}{\frac{\alpha}{n} - n}\right) = \exp(0) = 1$$