Hydrogen atom 2

In the previous section we used

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}$$

Alternatively, a_0 can be left undefined by using the Hamiltonian

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{\hbar^2}{\mu a_0 r}$$

For the new H the eigenvalues are

$$E_n = -\frac{1}{2\mu}\left(\frac{\hbar}{a_0}\right)^2\frac{1}{n^2}$$

Verify dimensions.

$$\frac{\hbar^2}{\mu a_0 r} = \frac{[\mathrm{J\,s}]^2}{[\mathrm{kg}]\,[\mathrm{m}]\,[\mathrm{m}]} = [\mathrm{J}]$$

$$\frac{1}{2\mu}\left(\frac{\hbar}{a_0}\right)^2\frac{1}{n^2} = \frac{1}{[\mathrm{kg}]}\frac{[\mathrm{J\,s}]^2}{[\mathrm{m}]^2} = [\mathrm{J}]$$

Eigenmath script