

Harmonic oscillator 3

From the previous section

$$a_+ = \sqrt{\frac{\hbar}{2m\omega}} \left(\frac{m\omega x}{\hbar} - \frac{d}{dx} \right)$$

$$a_- = \sqrt{\frac{\hbar}{2m\omega}} \left(\frac{m\omega x}{\hbar} + \frac{d}{dx} \right)$$

Substitute $ip/\hbar$ for d/dx .

$$a_+ = \sqrt{\frac{\hbar}{2m\omega}} \left(\frac{m\omega x}{\hbar} - \frac{ip}{\hbar} \right)$$

$$a_- = \sqrt{\frac{\hbar}{2m\omega}} \left(\frac{m\omega x}{\hbar} + \frac{ip}{\hbar} \right)$$

Factor out $\hbar$.

$$a_+ = \frac{1}{\sqrt{2m\hbar\omega}} (m\omega x - ip)$$

$$a_- = \frac{1}{\sqrt{2m\hbar\omega}} (m\omega x + ip)$$

Separate x and p .

$$a_+ + a_- = \frac{2}{\sqrt{2m\hbar\omega}} m\omega x$$

$$a_+ - a_- = -\frac{2}{\sqrt{2m\hbar\omega}} ip$$

Solve for x and p .

$$x = \sqrt{\frac{\hbar}{2m\omega}} (a_+ + a_-) \tag{1}$$

$$p = i\sqrt{\frac{m\hbar\omega}{2}} (a_+ - a_-) \tag{2}$$

From the previous section

$$a_+ \psi_j = \sqrt{j+1} \psi_{j+1}$$

$$a_- \psi_j = \sqrt{j} \psi_{j-1}$$

Hence

$$x \psi_j = \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{j+1} \psi_{j+1} + \sqrt{j} \psi_{j-1} \right)$$

$$p \psi_j = i\sqrt{\frac{m\hbar\omega}{2}} \left(\sqrt{j+1} \psi_{j+1} - \sqrt{j} \psi_{j-1} \right)$$

Eigenmath script