Harmonic oscillator 1

Let $V(x)$ be the potential energy for a harmonic oscillator.

$$V(x) = \frac{m\omega^2 x^2}{2}$$

We seek $\psi(x,t)$ that solves the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

The solution is

$$\psi_n(x,t) = C_n \exp\left[\frac{m\omega x^2}{2\hbar} - i\omega t\left(n + \tfrac{1}{2}\right)\right]\frac{d^n}{dx^n}\exp\left(-\frac{m\omega x^2}{\hbar}\right)$$

where $n = 0, 1, 2, \ldots$ and C_n is the normalization constant

$$C_n = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(\frac{\hbar}{m\omega}\right)^{n/2}$$

By inspection of the time exponential in $\psi_n(x,t)$ we find that

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hbar\omega\left(n + \tfrac{1}{2}\right)\psi(x,t)$$

Hence $\psi(x,t)$ also solves the time-independent Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = E_n\psi(x,t) \tag{1}$$

where for the harmonic oscillator

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

The time exponential of $\psi_n(x,t)$ cancels in (1) hence $\psi_n(x)$ is also a solution where

$$\psi_n(x) = C_n \exp\left(\frac{m\omega x^2}{2\hbar}\right)\frac{d^n}{dx^n}\exp\left(-\frac{m\omega x^2}{\hbar}\right)$$

By convention a factor of $(-1)^n$ is included.

$$\psi_n(x) = (-1)^n C_n \exp\left(\frac{m\omega x^2}{2\hbar}\right)\frac{d^n}{dx^n}\exp\left(-\frac{m\omega x^2}{\hbar}\right)$$

This factor cancels in the Schrodinger equation so it is not really needed.

Eigenmath script