

Galilean transformation

Let G_1 , G_2 , and G_3 be Galilean transformation operators.

$$G_1 = \frac{m}{\hbar}X_1, \quad G_2 = \frac{m}{\hbar}X_2, \quad G_3 = \frac{m}{\hbar}X_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon G_3 - \frac{1}{2}\epsilon^2 G_3^2$$

1. Show that to order ϵ^2

$$\begin{aligned}U^{-1}X_1U &= X_1 \\U^{-1}X_2U &= X_2 \\U^{-1}X_3U &= X_3\end{aligned}$$

2. Show that to order ϵ^2

$$\begin{aligned}U^{-1}P_1U &= P_1 \\U^{-1}P_2U &= P_2 \\U^{-1}P_3U &= P_3 - \epsilon m\end{aligned}$$

3. Show that to order ϵ^2

$$\begin{aligned}U^{-1}L_1U &= L_1 - \epsilon mX_1 \\U^{-1}L_2U &= L_2 + \epsilon mX_2 \\U^{-1}L_3U &= L_3\end{aligned}$$

4. Show that to order ϵ^2

$$U^{-1}HU = H - \epsilon P_3 + \frac{1}{2}\epsilon^2 m$$

where

$$H = \frac{1}{2m} (P_1^2 + P_2^2 + P_3^2)$$