

Fine structure 2

Let E_n be the Bohr energy for hydrogen.

$$E_n = -\frac{\mu c^2 \alpha^2}{2n^2}$$

Let E_r be the first order relativistic correction.

$$E_r = -\frac{E_n^2}{2\mu c^2} \left(\frac{4n}{l + \frac{1}{2}} - 3 \right)$$

Let E_{so} be the first order spin-orbit correction.

$$E_{so} = \frac{nE_n^2}{\mu c^2} \left(\frac{j(j+1) - l(l+1) - \frac{3}{4}}{l(l + \frac{1}{2})(l+1)} \right)$$

Show that

$$E_n + E_r + E_{so} = E_n \left(1 + \frac{\alpha^2}{n(j + \frac{1}{2})} - \frac{3\alpha^2}{4n^2} \right) \quad (1)$$

for $j = l \pm 1/2$.