Dirac spinor normalization

A widely-used convention is

$$u^\dagger u = 2E$$

where E is relativistic energy

$$E = \sqrt{\vec{p}\cdot\vec{p} + m^2}$$

For example, for a spin-up electron

$$u = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \dfrac{p_z}{E+m} \\ \dfrac{p_x + ip_y}{E+m} \end{pmatrix} \qquad u^\dagger u = 2E \tag{1}$$

To demonstrate Lorentz invariance, let $\vec{p} = 0$. Then $\sqrt{E+m} = \sqrt{2m}$ and

$$u = \sqrt{2m}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^\dagger u = 2m = 2E$$

It is instructive to see what happens when u is not normalized.

$$u = \begin{pmatrix} 1 \\ 0 \\ \dfrac{p_z}{E+m} \\ \dfrac{p_x + ip_y}{E+m} \end{pmatrix} \qquad u^\dagger u = \frac{\vec{p}\cdot\vec{p}}{(E+m)^2} + 1 \tag{2}$$

The issue in (2) is $u^\dagger u$ is not Lorentz invariant. For example, for $\vec{p} = 0$ we have

$$u^\dagger u = 1$$

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