# Dirac from boost

This is a Dirac spinor that represents an electron at rest with spin up along the $z$ axis.

$$u_0 = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

This matrix boosts a spinor in the $z$ direction where $E^2 = p^2 + m^2$.

$$\Lambda = \frac{1}{\sqrt{2m(E+m)}}\begin{pmatrix}E+m & 0 & p & 0\\ 0 & E+m & 0 & p\\ p & 0 & E+m & 0\\ 0 & p & 0 & E+m\end{pmatrix}$$

Hence

$$u = \Lambda u_0 = \frac{1}{\sqrt{E+m}}\begin{pmatrix}E+m & 0 & p & 0\\ 0 & E+m & 0 & p\\ p & 0 & E+m & 0\\ 0 & p & 0 & E+m\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \frac{1}{\sqrt{E+m}}\begin{pmatrix}E+m\\0\\p\\0\end{pmatrix}$$

This is the Dirac equation in spinor form.

$$\not{p}u = mu$$

Substitute $\Lambda u_0$ for $u$ on the right-hand side.

$$\not{p}u = m\Lambda u_0$$

Substitute $\gamma^0 u_0$ for $u_0$.

$$\not{p}u = m\Lambda\gamma^0 u_0$$

Substitute $\Lambda^{-1}u$ for $u_0$.

$$\not{p}u = m\Lambda\gamma^0\Lambda^{-1}u$$

Cancel $u$ to obtain

$$\not{p} = m\Lambda\gamma^0\Lambda^{-1}$$

Multiply both sides by $m^{-1}$ and $\Lambda$.

$$m^{-1}\not{p}\Lambda = \Lambda\gamma^0 \tag{1}$$

To recover the Dirac equation, start with this identity.

$$\gamma^0 u_0 = u_0$$

Boost both sides of the equation.

$$\Lambda\gamma^0 u_0 = \Lambda u_0$$

By equation (1) we have

$$m^{-1}\not{p}\Lambda u_0 = \Lambda u_0$$

Hence

$$\not{p}u = mu \tag{2}$$