Dirac equation 1

From Fitzpatrick problem 11.11.

Let

$$\psi^r = \exp\left(-\frac{i}{\hbar}\epsilon_r p_\mu x^\mu\right) w^r(\mathbf{p})$$

where $\epsilon = (1, 1, -1, -1)$ and

$$w^1(\mathbf{p}) = \sqrt{\frac{E + mc^2}{2mc^2}} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z c}{E+mc^2} \\ \frac{(p_x+ip_y)c}{E+mc^2} \end{pmatrix} \qquad w^2(\mathbf{p}) = \sqrt{\frac{E + mc^2}{2mc^2}} \begin{pmatrix} 0 \\ 1 \\ \frac{(p_x-ip_y)c}{E+mc^2} \\ \frac{-p_z c}{E+mc^2} \end{pmatrix}$$

$$w^3(\mathbf{p}) = \sqrt{\frac{E + mc^2}{2mc^2}} \begin{pmatrix} \frac{p_z c}{E+mc^2} \\ \frac{(p_x+ip_y)c}{E+mc^2} \\ 1 \\ 0 \end{pmatrix} \qquad w^4(\mathbf{p}) = \sqrt{\frac{E + mc^2}{2mc^2}} \begin{pmatrix} \frac{(p_x-ip_y)c}{E+mc^2} \\ \frac{-p_z c}{E+mc^2} \\ 0 \\ 1 \end{pmatrix}$$

Verify that ψ^r are solutions to the Dirac equation

$$i\hbar\gamma^\mu\partial_\mu\psi = mc\psi$$

and that the solutions are normalized as

$$|\psi|^2 = \frac{E}{mc^2}$$

Note that

$$\partial_0 = \frac{\partial}{\partial(ct)} = \frac{1}{c}\frac{\partial}{\partial t}$$

The gradient ∂_μ is a covariant vector hence

$$\gamma^\mu\partial_\mu = \frac{1}{c}\gamma^0\frac{\partial}{\partial t} + \gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}$$

Eigenmath script