Angular separation

Let P_1 and P_2 be points in polar coordinates such that

$$P_1 = r_1\hat{\mathbf{e}}_r + \theta_1\hat{\mathbf{e}}_\theta + \phi_1\hat{\mathbf{e}}_\phi, \quad P_2 = r_2\hat{\mathbf{e}}_r + \theta_2\hat{\mathbf{e}}_\theta + \phi_2\hat{\mathbf{e}}_\phi$$

The angular separation θ_{12} between P_1 and P_2 is

$$\begin{aligned}\cos\theta_{12} &= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)\\ &= \frac{r_1^2 + r_2^2 - r_{12}^2}{2r_1r_2}\\ &= \frac{x_1x_2 + y_1y_2 + z_1z_2}{r_1r_2}\end{aligned}$$

where

$$r_{12}^2 = (x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2$$

and

$$\begin{aligned} x_1 &= r_1\sin\theta_1\cos\phi_1 & \qquad x_2 &= r_2\sin\theta_2\cos\phi_2\\ y_1 &= r_1\sin\theta_1\sin\phi_1 & \qquad y_2 &= r_2\sin\theta_2\sin\phi_2\\ z_1 &= r_1\cos\theta_1 & \qquad z_2 &= r_2\cos\theta_2\end{aligned}$$