

# Angular momentum

Let

$$L_1 = X_2P_3 - X_3P_2$$

$$L_2 = X_3P_1 - X_1P_3$$

$$L_3 = X_1P_2 - X_2P_1$$

Show that

$$X_1L_3 - L_3X_1 = -i\hbar X_2$$

$$X_2L_3 - L_3X_2 = i\hbar X_1$$

$$X_3L_3 - L_3X_3 = 0$$

and

$$P_1L_3 - L_3P_1 = -i\hbar P_2$$

$$P_2L_3 - L_3P_2 = i\hbar P_1$$

$$P_3L_3 - L_3P_3 = 0$$

Let

$$L^2 = L_1^2 + L_2^2 + L_3^2$$

Show that

$$L_1L^2 - L^2L_1 = 0$$

$$L_2L^2 - L^2L_2 = 0$$

$$L_3L^2 - L^2L_3 = 0$$

Let

$$J_1 = \frac{1}{\hbar}L_1, \quad J_2 = \frac{1}{\hbar}L_2, \quad J_3 = \frac{1}{\hbar}L_3$$

Show that

$$J_1J_2 - J_2J_1 = iJ_3$$

$$J_2J_3 - J_3J_2 = iJ_1$$

$$J_3J_1 - J_1J_3 = iJ_2$$