Angular momentum 4

From the previous section we have quantum numbers l and m such that

$$l = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2, \ldots$$

and

$$m = -l, -l+1, \ldots, l-1, l$$

The eigenvalues of L^2 are $l(l+1)\hbar$.

$$L^2\psi = l(l+1)\hbar^2\psi$$

The eigenvalues of L_z are $m\hbar$.

$$L_z\psi = m\hbar\psi$$

We now seek eigenfunctions ψ that solve these eigenvalue equations.

For integer l the eigenfunctions are spherical harmonics Y_{lm}.

$$\psi = Y_{lm}(\theta, \phi)$$

Hence

$$L^2 Y_{lm} = l(l+1)\hbar^2 Y_{lm}$$

and

$$L_z Y_{lm} = m\hbar Y_{lm}$$

It turns out that L^2 and L_z have no eigenfunctions with half-integer eigenvalues.

Eigenmath script