Alpha decay 3

In this section we derive a regression formula for the following half-life data. Rows with $Z = 90$ are uranium and $Z = 88$ are thorium. Recall that Z is atomic number minus two.

$\log\tau_{1/2}$	A	Z	E
6.30	228	90	6.80
14.4	230	90	5.99
21.5	232	90	5.41
29.7	234	90	4.86
34.2	236	90	4.57
39.5	238	90	4.27
7.52	226	88	6.45
17.9	228	88	5.52
28.5	230	88	4.77
40.6	232	88	4.08

From the previous sections we have

$$\log\tau_{1/2} \propto \gamma \qquad \gamma = C_1 \frac{Z}{\sqrt{E}} - C_2\sqrt{Zr_1}$$

Using the approximations $r_1 \propto A^{1/3}$ and $A \propto Z$ we have

$$\sqrt{Zr_1} \propto \sqrt{ZZ^{1/3}} = Z^{2/3}$$

Hence the Geiger–Nuttall regression model

$$\log\tau_{1/2} = \beta_2 \frac{Z}{\sqrt{E}} + \beta_1 Z^{2/3} + \beta_0$$

By ordinary least squares the best coefficients for the above data are

$$\beta_2 = 3.702$$
$$\beta_1 = -3.104$$
$$\beta_0 = -59.28$$

The regression formula yields the following predicted values.

$\log\tau_{1/2}$	Predicted $\log\tau_{1/2}$
6.30	6.15
14.4	14.5
21.5	21.6
29.7	29.6
34.2	34.2
39.5	39.6
7.52	7.56
17.9	18.0
28.5	28.5
40.6	40.6

The observed linearity of $\log \tau_{1/2}$ and $Z/\sqrt{E}$ confirms the tunneling hypothesis for alpha decay.

Eigenmath script