Alpha decay 2

Let our hypothesis be that the nucleus produces alpha particles at an average rate ν which then tunnel with probability $e^{-2\gamma}$. Hence the decay rate λ is

$$\lambda = \nu e^{-2\gamma}$$

The half-life $\tau_{1/2}$ is

$$\tau_{1/2} = \frac{\log 2}{\lambda} = \frac{\log 2}{\nu} e^{2\gamma}$$

We seek to calculate $\tau_{1/2}$ from γ. The problem is that ν is only known empirically so we use the following modified Geiger-Nuttall formula

$$\log \tau_{1/2} = 3.682 \frac{Z}{\sqrt{E}} - 120.8$$

where $\tau_{1/2}$ is time in seconds and E is energy in MeV.

For the alpha decay process ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th}$ we have

$$Z = 90$$
$$E = 4.27\,\mathrm{MeV}$$

Hence

$$\tau_{1/2} = 1.49 \times 10^{17}\,\mathrm{s} = 4.72 \times 10^{9}\,\mathrm{yr}$$

The actual half-life of ${}^{238}\mathrm{U}$ is

$$4.46 \times 10^{9}\,\mathrm{yr}$$

Eigenmath script