Addition of angular momentum 3

Let Ψ be an eigenstate of J^2. Then for total angular momentum quantum number j

$$J^2\Psi = j(j+1)\hbar^2\Psi$$

The following Ψ are eigenstates with $j = l + \frac{1}{2}$.

$$\Psi = Y_{l,l}\chi_+$$
$$\Psi = Y_{l,-l}\chi_-$$

For $m < l$ the following linear combinations are eigenstates.

$$\Psi = \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_{lm}\chi_+ + \left(\frac{l-m}{2l+1}\right)^{1/2} Y_{l,m+1}\chi_- \qquad j = l + \tfrac{1}{2}$$
$$\Psi = \left(\frac{l-m}{2l+1}\right)^{1/2} Y_{lm}\chi_+ - \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_{l,m+1}\chi_- \qquad j = l - \tfrac{1}{2}$$