Consider a system with the following eigenstates.

$$
\begin{aligned}
|0\rangle &= (1\ 0\ 0\ 0)^\dagger && \text{no electrons} \\
|1\rangle &= (0\ 1\ 0\ 0)^\dagger && \text{one electron in state } \phi_1 \\
|2\rangle &= (0\ 0\ 1\ 0)^\dagger && \text{one electron in state } \phi_2 \\
|3\rangle &= (0\ 0\ 0\ 1)^\dagger && \text{two electrons, one in state } \phi_1, \text{ one in state } \phi_2
\end{aligned}
$$

Let electron states ϕ_n be modeled by a one dimensional box of length L.

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

Let $|\xi\rangle$ be an arbitrary normalized state vector.

$$|\xi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + c_3|3\rangle, \qquad \langle\xi|\xi\rangle = 1$$

Let us determine an energy matrix $\hat{E}$ such that the expected energy $\langle E\rangle$ in state $|\xi\rangle$ is

$$\langle E\rangle = \langle\xi|\hat{E}|\xi\rangle$$

Energy matrix $\hat{E}$ is the sum of kinetic and potential energy matrices.

$$\hat{E} = \hat{K} + \hat{V}$$

Kinetic energy matrix $\hat{K}$ can be computed from energy eigenvalues of the box model.

$$\hat{K} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E_1 & 0 & 0 \\ 0 & 0 & E_2 & 0 \\ 0 & 0 & 0 & E_1 + E_2 \end{pmatrix}, \qquad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Potential energy matrix $\hat{V}$ has one entry due to Coulomb interaction in the two electron state.

$$\hat{V} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V \end{pmatrix}$$

Let $\psi(x, y)$ be the antisymmetrized wavefunction of the two electrons.

$$\psi(x,y) = \frac{1}{\sqrt{2}}\big(\phi_1(x)\phi_2(y) - \phi_1(y)\phi_2(x)\big)$$

Then

$$V = \frac{e^2}{4\pi\epsilon_0} \int_0^L \int_0^L \psi^*(x,y) \left(\frac{1}{|x-y|}\right) \psi(x,y)\, dx\, dy$$

Let us now choose $L = 10^{-9}$ meters and compute numerical values. For $\hat{K}$ we have

$$\hat{K} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.38\,\text{eV} & 0 & 0 \\ 0 & 0 & 1.50\,\text{eV} & 0 \\ 0 & 0 & 0 & 1.88\,\text{eV} \end{pmatrix}$$

Computing V by numerical integration we have

$$\hat{V} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.67\,\mathrm{eV} \end{pmatrix}$$

Hence

$$\hat{E} = \hat{K} + \hat{V} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.38\,\mathrm{eV} & 0 & 0 \\ 0 & 0 & 1.50\,\mathrm{eV} & 0 \\ 0 & 0 & 0 & 6.55\,\mathrm{eV} \end{pmatrix}$$