Consider the following eigenstates of a hypothetical quantum system.[1]

$\|00\rangle = (1,0,0,0)$	no fermions
$\|10\rangle = (0,1,0,0)$	one fermion in state 1
$\|01\rangle = (0,0,1,0)$	one fermion in state 2
$\|11\rangle = (0,0,0,1)$	two fermions, one in state 1, one in state 2

Creation and annihilation operators are formed from outer products of state vectors. Sign changes make the operators antisymmetric.

$\hat{b}_1^\dagger = \|10\rangle\langle 00\| - \|11\rangle\langle 01\|$	Create one fermion in state 1
$\hat{b}_1 = \|00\rangle\langle 10\| - \|01\rangle\langle 11\|$	Annihilate one fermion in state 1
$\hat{b}_2^\dagger = \|01\rangle\langle 00\| + \|11\rangle\langle 10\|$	Create one fermion in state 2
$\hat{b}_2 = \|00\rangle\langle 01\| + \|10\rangle\langle 11\|$	Annihilate one fermion in state 2

The operators in matrix form.

$$\hat{b}_1^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad \hat{b}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \hat{b}_2^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \hat{b}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Verify anticommutation relations of the operators.

$$\hat{b}_j \hat{b}_k + \hat{b}_k \hat{b}_j = 0$$

$$\hat{b}_j^\dagger \hat{b}_k^\dagger + \hat{b}_k^\dagger \hat{b}_j^\dagger = 0$$

$$\hat{b}_j \hat{b}_k^\dagger + \hat{b}_k^\dagger \hat{b}_j = \delta_{jk}$$

[1] Adapted from problem 16.1.1 of "Quantum Mechanics for Scientists and Engineers."
`https://ee.stanford.edu/~dabm/QMbook.html`